

2012年度 第24回の整数論セミナー

日時： 2012年12月14日(金) 16:30 ~ 18:00

場所： 〒169-8555 東京都新宿区大久保3-4-1
早稲田大学西早稲田キャンパス (旧・大久保キャンパス)
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講演者： Chantal David (Concordia University)

タイトル: Elliptic curves with prescribed groups over finite fields and the Cohen-Lenstra Heuristics

アブストラクト：

Let $G_{m,k} := \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/mk\mathbb{Z}$ be an abelian group of rank 2 and order $N = mk^2$.

When does there exist a finite field \mathbb{F}_p and an elliptic curve E/\mathbb{F}_p such that $E(\mathbb{F}_p) \simeq G_{m,k}$?

We show that this happens with probability 0 when k is very small with respect to m , and with probability 1 when k is big enough with respect to m .

The fact that the groups $G_{m,k}$ are more likely to occur when k is big is reminiscent of the Cohen-Lenstra heuristics which predict that a random abelian group G occurs with probability weighted by $\#G/\#\text{Aut}(G)$.

By counting the average number of times that a given group $G_{m,k}$ occurs over the finite fields \mathbb{F}_p (and not simply if a given group occurs or not), we are able to verify that the probability of occurrence of the groups $G_{m,k}$ is indeed weighted by the Cohen-Lenstra weights.

This is joint work with V. Chandee, D. Koukoulopoulos and E. Smith.