Waseda Number Theory Workshop 2024

Titles and Abstracts (English)

March 5th(Tuesday)

March 5th, 10:40–11:40 **Keiji Okano (Tsuru University)**

Title: On the triviality of the unramified Iwasawa modules of the maximal multiple $\mathbb{Z}_p$-extensions.

Abstract:
For a number field k, we consider the maximal multiple $\mathbb{Z}_p$-extension $\tilde{k}$ and the unramified Iwasawa module $X(\tilde{k})$, which is the Galois group of the maximal unramified abelian p-extension over $\tilde{k}$. There is a conjecture that $X(\tilde{k})$ would be pseudo-null as a $\mathbb{Z}_p[[\mathrm{Gal}(\tilde{k}/k)]]$-module. Roughly speaking, it means that $X(\tilde{k})$ is quite small. So, a question arises: when does $X(\tilde{k})$ become trivial? In more detail, the aim of this study is, for an odd prime p, to classify CM-fields k which are decomposed completely at p such that $X(\tilde{k}) = 0$. We find that the degree of k should be equal or less than 6. In this talk, I will introduce a result in the case where the degree of k is 4.

March 5th, 12:00–13:00 **Hiroshi Ishimoto (Osaka Metropolitan University)**

Title: Arthur's classification for non-quasi-split odd special orthogonal groups.

Abstract:
In 2013, Arthur established a classification of discrete automorphic representations of quasi-split orthogonal and symplectic groups. We shall call it Arthur's classification, or Arthur's multiplicity formula. Following the work, Mok analogously proved it for quasi-split unitary groups. Later, Kaletha-Minguez-Shin-White gave the classification for non-quasi-split unitary groups. We can prove it for non-quasi-split odd special orthogonal groups by the method of Kaletha-Minguez-Shin-White. In this talk, I would like to explain the statement and the method of Kaletha-Minguez-Shin-White.

March 5th, 14:30–15:30 **Hirotaka Kakuhama (Hokkaido University)**

Title: Local theta correspondences and Langlands parameters.

Abstract:
The local theta correspondences give a method of construction of irreducible representations of certain groups from irreducible representations of other groups. Although it is defined in an analytic way, it is known that the local theta correspondences for $(\mathrm{Sp}, \mathrm{O})$-pairs and (U, U)-pairs are described in simple manners in terms of Langlands parameters. In this talk, I explain some backgrounds and progress of the research of local theta correspondence and Langlands parameters. In particular, we formulate the description of the local theta correspondence for pairs of quaternionic unitary groups.

March 5th, 15:50–16:50 **Yasuhiro Oki (Hokkaido University)**

Title: The Hasse norm principle for some non-Galois extensions of square-free degree.

Abstract:
The Hasse norm principle is one of the classical problem in algebraic number theory, which asserts that the norms of finite extensions of number fields can be described by their local norms. If d is a square-free positive integer, Gurak and Endo–Miyata has been proved the validity of the Hasse norm principle for Galois extensions of degree d. However, it is generally unresolved whether the same argument as above holds when the Galois condition is removed. In this talk, we give an existence of the failure of the Hasse norm principle for finite extensions of degree d of number fields, where d is a square-free composite number that is a multiple of 3.

March 5th, 17:10–18:10 **Axel Kleinschmidt (Max Planck Institute for Gravitational Physics)**

Title: String scattering amplitudes and small representations.

Abstract:
One of the basic objects of interest in string theory is a scattering amplitude that captures the interactions between different strings. It turns out that, in certain contexts, such scattering amplitudes enjoy automorphic properties. The groups appearing in this connection are split real groups of different ranks and also include exceptional groups. I will review this phenomenon and explain what types of automorphic representations arise and what physical information is encoded in them. Extrapolating the way these automorphic representations arise in string theory suggests a more general class of automorphic representations where the Z-finiteness property is not satisfied.

March 6th(Wednesday)

March 6th, 10:40-11:40 **Tsuyoshi Itoh (Chiba Institute of Technology)**

Title: Consideration on $\mathbb{Z}_p$-extensions over the cyclotomic $\mathbb{Z}_p$-extension field of an algebraic number field.

Abstract:
Let k^c be the cyclotomic $\mathbb{Z}_p$-extension field of an algebraic number field k. We consider a $\mathbb{Z}_p$-extension K/k^c. (Note that K/k does not have to be a Galois extension.) I will introduce a result on the behavior of the p-part of class numbers of certain intermediate fields of K/k. From this result, we can obtain a result concerning the existence of tamely ramified cyclic extensions of p-power degree whose p-part of class numbers of intermediate fields behaves like Iwasawa's class number formula. To obtain examples of the above result, it is important to know the structure of the unramified Iwasawa module $X(K)$ as a $\mathbb{Z}_p[[\mathrm{Gal}(K/k^c)]]$-module. I will state several results on the structure of $X(K)$.

March 6th, 12:00-13:00 **Toshiyuki Takahashi (Waseda University)**

Title: Estimation of the number of the cyclic cubic fields whose class numbers are divisible by n.

Abstract:
Class number is an important object in the number theory. Especially, it is an interesting problem to estimate the number of algebraic number fields whose class numbers are divisible by n. In this talk, I will give an estimation of the number of the cyclic cubic fields whose class numbers are divisible by n, using the construction of such cubic cyclic fields by K. Uchida.

March 6th, 14:30-15:30 **Manabu Ozaki (Waseda University)**

Title: A number field analogue of Weil's theorem on congruent zeta functions.

Abstract:
Let K be a function field of one variable over a finite field $\mathbb{F}$. Weil's celebrated theorem states that the congruent zeta function of $K/\mathbb{F}$ is determined by the $\mathrm{Gal}(\overline{\mathbb{F}}/\mathbb{F})$-module structure of $X_{\overline{\mathbb{F}}K}(p) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$, and vise versa, where p is a prime number different from the characteristic of K and $X_{\overline{\mathbb{F}}K}(p)$ stands for the Galois group of the maximal unramified abelian p-extension over $\overline{\mathbb{F}}K$. In this talk I will give a number field analogue of the above mentioned theorem by considering the total cyclotomic extension, which we may regard as an analogue of $\overline{\mathbb{F}}K/K$.

March 6th, 15:50-16:50 **Kengo Fukunaga (Tokyo Institute of Technology)**

Title: Multi-variable p-adic power series of logarithmic order.

Abstract:
In 1975, Amice-Vélu introduced one-variable p-adic power series of logarithmic order and constructed one-variable p-adic L-functions attached to non-ordinary cusp forms.

By Vishik, Perrin-Riou and so on, the theory of one-variable p-adic power series of logarithmic order plays important roles in the theory of one-variable p-adic L-functions attached to non-ordinary cusp forms.

In this talk, we introduce multi-variable p-adic power series of logarithmic order and present our results which I believe are essential for the theory of multi-variable p-adic L-functions attached to non-ordinary cusp forms.

This is a joint work with Tadashi Ochiai (Tokyo Institute of Technology).

March 6th, 17:10-18:10 **Shoyu Nagaoka (Kinki University)**

Title: On $\boldsymbol{p}$-divisivility of Fourier coefficients of Siegel modular forms.

Abstract:
We describe the p-divisibility transposition for the Fourier coefficients of Siegel modular forms.
Let F be a Siegel modular form with the Fourier expansion $F(Z) = \sum_T a(F,T)q^T$ whose Fourier coefficients $a(F,T)$ are p-integral. We schematize the following p-divisiblity transpositions:

$$(\mathrm{I})_p \quad \boxed{\text{All } a(F,T^{(n)}) \text{ for } T^{(n)} > 0 \text{ are divisible by } p.}$$
$$\Downarrow \ \Phi\text{: Siegel operator}$$
$$(\mathrm{II})_p \quad \boxed{\text{“Most” } a(\Phi(F),T^{(n-1)}) \text{ for } T^{(n-1)} > 0 \text{ are divisible by } p.}$$
$$\Downarrow \ \Phi\text{: Siegel operator}$$
$$(\mathrm{III})_p \quad \boxed{\text{Certain “special” } a(\Phi^2(F),T^{(n-2)}) \text{ for } T^{(n-2)} > 0 \text{ are divisible by } p.}$$

We will provide examples of modular forms that fit each of these stages by means of the Siegel Eisenstein series.

March 7th(Thursday)

March 7th, 9:10-10:10 **Arata Minamide (Research Institute for Mathematical Sciences Kyoto University)**

Title: On the essential logical structure of inter-universal Teichmüller theory I.

Abstract:
In this series of three talks, we will give a brief review of various important notions appearing in inter-universal Teichmüller theory and discuss the essential logical structure of the theory.

March 7th, 10:20-11:20 **Arata Minamide (Research Institute for Mathematical Sciences Kyoto University)**

Title: On the essential logical structure of inter-universal Teichmüller theory II.

Abstract:
This lecture follows Lecture I.

March 7th, 11:30–12:30 **Arata Minamide (Research Institute for Mathematical Sciences Kyoto University)**

Title: On the essential logical structure of inter-universal Teichmüller theory III..

Abstract:
This lecture follows Lecture I and Lecture II.

March 7th, 14:30-15:30 **Naoki Kumakawa (Waseda University)**

Title: A weak form of Greenberg's conjecture for the cyclotomic $\mathbb{Z}_2$-extension of real quadratic fields.

Abstract:
Let l be a prime number and K_∞/K the cyclotomic $\mathbb{Z}_l$-extension of a number field K. We denote by L_∞/K_∞ the maximal unramified abelian l-extension and let $X_\infty = \mathrm{Gal}(L_\infty/K_\infty)$. It is well known in Iwasawa theory that X_∞ is a finitely generated torsion $\mathbb{Z}_l[[\mathrm{Gal}(K_\infty/K)]]$-module. Greenberg's conjecture claims that if K is totally real, then X_∞ is a finite $\mathbb{Z}_l[[\mathrm{Gal}(K_\infty/K)]]$-module. On the other hand, there is also a weak version of this conjecture. The conjecture states that if K is totally real and $X_\infty \neq 0$, then there exists a non-trivial finite $\mathbb{Z}_l[[\mathrm{Gal}(K_\infty/K)]]$-submodule of X_∞. This is often called a weak form of Greenberg's conjecture. In this talk, we show that the weak form of Greenberg's conjecture is true if $l = 2$ and $K = \mathbb{Q}(\sqrt{p})$, where p is an odd prime number satisfying certain arithmetic conditions. As an application of this result, we obtain an upper bound of the Iwasawa λ-invariant of the cyclotomic $\mathbb{Z}_2$-extension of K under some additional conditions on K.

March 7th, 15:50-16:50 **Naoki Kumakawa (Waseda University), Yasushi Mizusawa (Rikkyo University)**

Title: On $\mathbb{Z}_2$-extensions unramified outside $(1 + \sqrt{-7})/2$.

Abstract:
The imaginary quadratic field $\mathbb{Q}(\sqrt{-7})$ of discriminant -7 has a non-cyclotomic $\mathbb{Z}_2$-extension unramified outside only one prime, which can be constructed by division points of an elliptic curve. Regarding such a $\mathbb{Z}_2$-extension as an analogue of the cyclotomic $\mathbb{Z}_2$-extension of rationals, we consider an analogous problem of Greenberg's conjecture for such non-cyclotomic $\mathbb{Z}_2$-extensions which are lifted over quadratic extensions of $\mathbb{Q}(\sqrt{-7})$. In particular, we give an analogue of a basic theorem of Ozaki and Taya on Greenberg's conjecture for cyclotomic $\mathbb{Z}_2$-extensions of real quadratic fields.

March 7th, 17:10-18:10 **Kazuma Shimomoto (Tokyo Institute of Technology)**

Title: Perfectoid spaces and applicatons to Noetherian rings.

Abstract:
Abstract: Some important conjectures in commutative algebra have been solved by the method of perfectoid spaces. In this talk, I will explain how perfectoid geometry is used to understand Noetherian rings of mixed characteristic through certain non-Noetherian rings along with recent results in this research.