Waseda Number Theory Workshop 2025

Titles and Abstracts (English)

March 13th(Thursday)

March 13th, 10:00–11:00 **Yusuke Nemoto (Waseda University Honjo Senior High School, Chiba University)**

Title: Non-torsion algebraic cycles on the Jacobians of Fermat quotients.

Abstract:
Ceresa cycles are important examples of algebraic cycles that are generically homologically trivial but algebraically non-trivial. However, it is difficult to show the non-triviality or non-torsioness for Ceresa cycles of specific curves. Regarding the algebraic non-triviality or non-torsioness of Ceresa cycles of Fermat curves and their quotients, there are some results by Harris, Bloch, Kimura, Tadokoro and Otsubo. Recently, Eskandari-Murty proved that the Ceresa cycle of the Fermat curve, whose degree is divisible by a prime greater than 7, is non-torsion modulo rational equivalence. In this talk, we prove that the Ceresa cycles of Fermat quotient curves are non-torsion modulo rational equivalence under some assumptions.

March 13th, 11:20–12:20 **Florian Sprung (Arizona State University)**

Title: What does the constant term of a characteristic power series know?

Abstract:
One important idea in Iwasawa theory is the following: We may try to understand an arithmetic object of interest by considering its behavior in the cyclotomic $\mathbb{Z}_p$-extension of a number field F. This behavior can be packaged into a cohomological group such as a Selmer group. Under nice circumstances, this Selmer group reveals much of its structure via its characteristic power series $f(X)$. When evaluated at $X = 0$, this power series should know the arithmetic object in the number field F we started with. We present some results on this idea in the context of elliptic curves and in the case of modular forms. Some of the results presented are joint work with Jishnu Ray.

March 13th, 14:00–15:00 **Yuta Kadono (Tohoku University)**

Title: On the Hurwitz-Lerch type central binomial series.

Abstract:
The central binomial series (CBS) is a type of Dirichlet series that features a central binomial coefficient in the summand. The special values at integer points in CBS have some interesting properties. A notable result is an explicit formula by Lehmer using two polynomials with integer coefficients, an arcsine function, and a radical function for the special values at negative integer points of the CBS. Furthermore, Bényi-Matsusaka showed that these polynomials are essentially bivariate Eulerian polynomials and that certain special values of polynomials can be expressed in terms of poly-Bernoulli numbers. In this talk, we introduce Hurwitz type CBS (HCBS, CBS with one more real parameter) and present some properties of special values at negative integer points. In particular, we give an explicit formula using polynomials with integer coefficients and special functions for the special values at negative integer points of the Hurwitz-Lerch type CBS (one variable functionalisation of HCBS), similar to the classical CBS. It is further shown that the polynomials appearing in this explicit formula are again related to the bivariate Eulerian polynomials. This talk is based on ongoing joint work with Karin Ikeda from Kyushu University.

March 13th, 15:20–16:20 **Yusaku Nishimura (Waseda University)**

Title: The applications of characters over a finite field to graph theory
– The universality of Paley graphs –.

Abstract:
There are many applications of number theory over finite fields in graph theory. In the first part of this talk, we introduce the results on the universality of Paley graphs, which are obtained using Weil's bound for character sums over finite fields. In the last part of this talk, we introduce the (complete) invariants for graphs and discuss their properties. This talk is a collaborative research effort with Miezaki, Munemasa, Sakuma, and Tsujie.

March 13th, 16:40–17:40 **Koji Tasaka (Kindai University)**

Title: Spherical T-design and modular forms for quaternions.

Abstract:
In this talk, we will explore a spherical design-theoretic property of finite subsets of the unit sphere, based on a quantity known as 'strength.' When the finite set is derived as a shell of a lattice, determining this strength is closely related to the theory of modular forms, as highlighted in the works of Venkov (1985) and others. I will discuss the connection between spherical T-designs, a slightly extended concept of spherical designs, and modular forms. I will focus on the case of quaternions. This is joint work with Hiroshi Nozaki (Aichi University of Education) and Masatake Hirao (Aichi Prefectural University).

March 14th(Friday)

March 14th, 10:00–11:00 **Kazuki Tomiyama (Waseda University)**

Title: Arithmetic properties of singular values of Hauptmoduln.

Abstract:
The classical theory of the elliptic modular j function provides remarkable properties of its CM values (singular moduli), which play a crucial role in number theory and the theory of elliptic curves. In another aspect, the j function is related to sporadic finite simple groups, in the context of Moonshine. Each conjugacy class of the Monster group, the largest sporadic simple group, produces a modular function with special properties, called a Hauptmodul. These modular functions arising from the Monster group are called McKay-Thompson series, and the j function is the function obtained from the identity class of the Monster. Chen-Yui proved that the CM values of some McKay-Thompson series are algebraic integers. In this talk, we extend these results to Hauptmoduln with cyclotomic integer Fourier coefficients, by using generalized modular equations.

March 14th, 11:20–12:20 **John Duncan (Academia Sinica)**

Title: Arithmetic, Modular Forms, and Modules for the Smallest Mathieu Group.

Abstract:
One of the main ideas motivating monstrous moonshine is the notion that the most "natural" representation of the monster group is infinite dimensional, even though the monster itself is finite. In this talk we will present some infinite-dimensional representations of the smallest of the sporadic simple groups, and explain how they connect, via modular forms, to some questions in elliptic curve arithmetic.

March 14th, 14:00–15:00 **Kazuya Kawasetsu (Kumamoto University)**

Title: Rogers-Ramanujan exact sequences and free modules over free generalized vertex algebras.

Abstract:
In this talk, we introduce notion of free modules over (generalized) vertex algebras. A series of recursion formulas, which generalizes a classical formula used to prove the famous Rogers-Ramanujan (RR) identities and RR continued fraction formula, is conceptually obtained from short exact sequences among free modules over free generalized vertex algebras, the RR exact sequences. They include as a special case one equivalent to the exact sequence constructed by S. Capparelli et al. using intertwining operators in the theory of vertex operator algebras. We give applications of the RR exact sequences to representation theory and related topics.

March 14th, 15:20–16:20 **Toshiki Matsusaka (Kyushu University)**

Title: Eichler–Selberg relations for singular moduli.

Abstract:
The Eichler–Selberg trace formula expresses the trace of Hecke operators on spaces of cusp forms as weighted sums of Hurwitz–Kronecker class numbers. We extend this formula to a natural class of relations for traces of singular moduli, where one views class numbers as traces of the constant function $j_0(\tau) = 1$. More generally, we consider the singular moduli for the Hecke system of modular functions $j_m(\tau) := mT_m(j(\tau) - 744)$. For each $\nu \geq 0$ and $m \geq 1$, we obtain an Eichler–Selberg relation. For $\nu = 0$ and $m \in \{1, 2\}$, these relations are Kaneko's celebrated singular moduli formulas for the coefficients of $j(\tau)$. For each $\nu \geq 1$ and $m \geq 1$, we obtain a new Eichler–Selberg trace formula for the Hecke action on the space of weight $2\nu + 2$ cusp forms, where the traces of $j_m(\tau)$ singular moduli replace Hurwitz–Kronecker class numbers. These formulas involve a new term that is assembled from values of symmetrized shifted convolution L-functions. (This is a joint work with Yuqi Deng and Ken Ono).

March 14th, 16:40–17:40 **Hisashi Kojima (Saitama University)**

Title: On the Ikeda conjecture concerning the period of Ikeda-Miyawaki lift of Siegel modular forms.

Abstract:
In 2001, T. Ikeda constructed the Ikeda lift from an elliptic cusp form to a Siegel cusp form of even degree. This is a higher dimensional version of Saito-Kurokawa lift of Siegel modular forms of degree two. In the Duke Math. J., 131 (2006), using the pullback of the Ikeda lift, he defined the Ikeda-Miyawaki lift from a Siegel cusp form f to a Siegel cusp form F. There he determined explicitly the standard L-function associated with F. Moreover, he proposed an interesting conjecture about a relation between the period of F and the product of several critical values of L-functions. Today, I don't know of any real-world examples of this conjecture.

In this lecture, we give an affirmative solution about this conjecture with week meaning under some conditions . Moreover, we confirm this conjecture rigorously in some cases. This is a joint work with T. Ibukiyama and H. Katsurada.

March 15th(Saturday)

March 15th, 10:00–11:00 **Nozomu Suzuki (Tokyo University of Science)**

Title: Calculating the index of equation orders using Newton polygons and its applications.

Abstract:
The gcd of the group index of equation orders in the ring of integers of a number field K is called the index of K. It is an obstruction of the integer ring to have a power basis and also is a product of primes to which Dedekind's prime decomposition theorem cannot directly applies. Montes and Nart gave a necessary and sufficient condition that the index of the equation order coincides with the quantity derived from the Newton polygon by extending a result of Ore. However, they omitted the proof of the result, and the condition has a minor gap. In this talk, a precise form of the theorem and its application will be given.

March 15th, 11:20–12:20 **Shunsuke Usuki (Kyoto University)**

Title: On a lower bound of the number of integers in Littlewood's conjecture.

Abstract:
Littlewood's conjecture is a famous and long-standing open problem in Diophantine approximation, and is closely related to the action of diagonal matrices on $\mathrm{SL}(3,\mathbb{R})/\mathrm{SL}(3,\mathbb{Z})$ (diagonal action). From this relation and a rigidity property of the action, a breakthrough for this conjecture was made in the 2000's. In this talk, I will explain such a relation between Littlewood's conjecture and the diagonal action. I will also explain my result on "quantitative" Littlewood's conjecture which is also derived from some properties of the diagonal action.

March 15th, 14:00–15:00 **Joseph Muller (The University of Tokyo)**

Title: Nearby cycles of the PEL $\mathrm{GU}(1, n-1)$ Shimura variety over a ramified prime.

Abstract:
Shimura varieties associated to groups of unitary similitudes have a classical description as moduli spaces of polarized abelian varieties with extra structures. In the case of signature $(1, n-1)$, Pappas built a flat integral model of this Shimura variety over primes which ramify in the reflex field. This model has isolated singularities located inside the special fiber. In this talk, I will explain how to compute the cohomology sheaves of the ℓ-adic nearby cycles complex of this integral model. Our computations rely on Krämer's explicit description of the local model and of its blow-up at the singular points. Since the blow-up has semi-stable reduction, the nearby cycles can now be computed by proper base change.

March 15th, 15:20–16:20 **Satoshi Fujii (Shimane University)**

Title: On families of imaginary abelian fields with pseudo-null unramified Iwasawa modules.

Abstract:
Let p be a prime number and k a finite extension of $\mathbb{Q}$. Let K/k be the maximal multiple $\mathbb{Z}_p$-extension and X the Galois group of the maximal unramified pro-p abelian extension L_K/K. Then the complete group ring $\Lambda = \mathbb{Z}_p[[Gal(K/k)]]$ acts on X, and it is known that X is finitely generated and torsion over Λ. Further, it is conjectured by Greenberg that X is pseudo-null over Λ. In this talk, for each prime number p, we shall show that there are infinitely many abelian fields k such that $X \neq 0$ and that X is pseudo-null.

March 15th, 16:40–17:40 **Norio Adachi （Waseda University）**

Title: The fundamental Equality for Semi-discrete Valuations.

Abstract:
Let v be a semi-discrete valuation of a field K, which indicates that the valuation group of v is isomorphic to the finite direct sum of the integers with lexicographic order. Let L be a finite separable extension of K. We study the conditions under which the fundamental equality of the extensions of v to L holds.