

2025 早稲田整数論研究集会

題目とアブストラクト (日本語版)

3 月 13 日 (木)

3 月 13 日, 10:00–11:00 根本 裕介 (早稲田大学本庄高等学院、千葉大学)

題目: Non-torsion algebraic cycles on the Jacobians of Fermat quotients.

アブストラクト:

Ceresa cycles are important examples of algebraic cycles that are generically homologically trivial but algebraically non-trivial. However, it is difficult to show the non-triviality or non-torsionness for Ceresa cycles of specific curves. Regarding the algebraic non-triviality or non-torsionness of Ceresa cycles of Fermat curves and their quotients, there are some results by Harris, Bloch, Kimura, Tadokoro and Otsubo. Recently, Eskandari-Murty proved that the Ceresa cycle of the Fermat curve, whose degree is divisible by a prime greater than 7, is non-torsion modulo rational equivalence. In this talk, we prove that the Ceresa cycles of Fermat quotient curves are non-torsion modulo rational equivalence under some assumptions.

3 月 13 日, 11:20–12:20 **Florian Sprung (Arizona State University)**

題目: 特性冪級数の定数項の情報について.

アブストラクト:

岩澤理論におけるアイデアの一つは、次のようなものです:

数論的対象を研究するには、対象を (ある固定された) 代数体 F の円分 \mathbb{Z}_p 拡大で「変形」するのが便利で、その変形の振る舞いをコホモロジーに現れる群 (セルマー群) で「はかる」ことができます。セルマー群の最も重要な不変量は特性冪級数 $f(X)$ であり、変数 X は変形に対応しています。 $f(X)$ の定数項 ($X = 0$ での情報) は変形の基底となる代数体 F での数論的対象の振る舞いを把握できるはずだと考えられます。

数論的対象が楕円曲線または保形形式の場合においていくつかの結果を紹介したいと思います。(主に J.Ray 氏との共同研究)。

3月13日, 14:00–15:00 角野 裕太 (東北大学)

題目: On the Hurwitz-Lerch type central binomial series.

アブストラクト:

The central binomial series (CBS) is a type of Dirichlet series that features a central binomial coefficient in the summand. The special values at integer points in CBS have some interesting properties. A notable result is an explicit formula by Lehmer using two polynomials with integer coefficients, an arcsine function, and a radical function for the special values at negative integer points of the CBS. Furthermore, Bényi-Matsusaka showed that these polynomials are essentially bivariate Eulerian polynomials and that certain special values of polynomials can be expressed in terms of poly-Bernoulli numbers. In this talk, we introduce Hurwitz type CBS (HCBS, CBS with one more real parameter) and present some properties of special values at negative integer points. In particular, we give an explicit formula using polynomials with integer coefficients and special functions for the special values at negative integer points of the Hurwitz-Lerch type CBS (one variable functionalisation of HCBS), similar to the classical CBS. It is further shown that the polynomials appearing in this explicit formula are again related to the bivariate Eulerian polynomials. This talk is based on ongoing joint work with Karin Ikeda from Kyushu University.

3月13日, 15:20–16:20 西村 優作 (早稲田大学)

題目: 有限体上の指標のグラフ理論への応用

– Paley グラフの普遍グラフ性について –.

アブストラクト:

有限体上の整数論は、グラフ理論への多くの応用が知られている。本講演前半では、有限体上の指標和に関する Weil の公式を用いた Paley グラフの普遍グラフ性についての結果を紹介する。後半では、普遍グラフ性を用いたグラフの（完全）不変量の構成、その不変量の性質を議論する。本講演は、三枝崎氏、宗政氏、佐久間氏、辻栄氏との共同研究である。

3月13日, 16:40–17:40 田坂 浩二 (近畿大学)

題目: Spherical T-design and modular forms for quaternions.

アブストラクト:

In this talk, we will explore a spherical design-theoretic property of finite subsets of the unit sphere, based on a quantity known as 'strength.' When the finite set is derived as a shell of a lattice, determining this strength is closely related to the theory of modular forms, as highlighted in the works of Venkov (1985) and others. I will discuss the connection between spherical T-designs, a slightly extended concept of spherical designs, and modular forms. I will focus on the case of quaternions. This is joint work with Hiroshi Nozaki (Aichi University of Education) and Masatake Hirao (Aichi Prefectural University).

3月14日, 10:00–11:00 富山 和樹 (早稲田大学)

題目: Hauptmodul の singular value の数論的性質.

アブストラクト:

数論や楕円曲線の理論を契機に古くから研究されてきた楕円モジュラー j 関数は、上半平面の虚 2 次点 (CM 点) において代数的整数の値をとり、その値は対応する虚 2 次体の Hilbert 類体を生成する．一方でいわゆる Moonshine を通して、位数最大の散在型有限単純群である Monster 群の各共役類から、Hauptmodul とよばれる特別な性質をもつ保型関数が得られ (McKay-Thompson 級数)、単位元に対応する Hauptmodul としては楕円モジュラー j 関数が得られる．Chen-Yui は一部の McKay-Thompson 級数の CM 点での値が代数的整数となることを証明した．本講演では、一般化されたモジュラー方程式を議論することで、この結果を円分体の整数環に Fourier 係数をとる Hauptmodul へと拡張した研究を紹介する．

3月14日, 11:20–12:20 John Duncan (Academia Sinica)

題目: Arithmetic, Modular Forms, and Modules for the Smallest Mathieu Group.

アブストラクト:

One of the main ideas motivating monstrous moonshine is the notion that the most "natural" representation of the monster group is infinite dimensional, even though the monster itself is finite. In this talk we will present some infinite-dimensional representations of the smallest of the sporadic simple groups, and explain how they connect, via modular forms, to some questions in elliptic curve arithmetic.

3月14日, 14:00–15:00 川節 和哉 (熊本大学)

題目: Rogers-Ramanujan exact sequences and free modules over free generalized vertex algebras.

アブストラクト:

In this talk, we introduce notion of free modules over (generalized) vertex algebras. A series of recursion formulas, which generalizes a classical formula used to prove the famous Rogers-Ramanujan (RR) identities and RR continued fraction formula, is conceptually obtained from short exact sequences among free modules over free generalized vertex algebras, the RR exact sequences. They include as a special case one equivalent to the exact sequence constructed by S. Capparelli et al. using intertwining operators in the theory of vertex operator algebras. We give applications of the RR exact sequences to representation theory and related topics.

3月14日, 15:20–16:20 松坂 俊輝 (九州大学)

題目: Eichler–Selberg relations for singular moduli.

アブストラクト:

The Eichler–Selberg trace formula expresses the trace of Hecke operators on spaces of cusp forms as weighted sums of Hurwitz–Kronecker class numbers. We extend this formula to a natural class of relations for traces of singular moduli, where one views class numbers as traces of the constant function $j_0(\tau) = 1$. More generally, we consider the singular moduli for the Hecke system of modular functions $j_m(\tau) := mT_m(j(\tau) - 744)$. For each $\nu \geq 0$ and $m \geq 1$, we obtain an Eichler–Selberg relation. For $\nu = 0$ and $m \in \{1, 2\}$, these relations are Kaneko’s celebrated singular moduli formulas for the coefficients of $j(\tau)$. For each $\nu \geq 1$ and $m \geq 1$, we obtain a new Eichler–Selberg trace formula for the Hecke action on the space of weight $2\nu + 2$ cusp forms, where the traces of $j_m(\tau)$ singular moduli replace Hurwitz–Kronecker class numbers. These formulas involve a new term that is assembled from values of symmetrized shifted convolution L -functions. (This is a joint work with Yuqi Deng and Ken Ono).

3月14日, 16:40–17:40 小嶋 久祉 (埼玉大学)

題目: ジーゲルモジュラー形式の池田–宮脇リフトの周期に関する池田予想について.

アブストラクト:

2001年に池田氏は次数2のジーゲルモジュラー形式の齋藤–黒川リフトの高次元版である偶数次数のジーゲルモジュラー形式への池田リフトを具体的に構成した。更に「Duke Math. J., 131 (2006)」において、この池田リフトの pullback を用いて、ジーゲルモジュラー形式 f からジーゲルモジュラー形式 F への池田–宮脇リフトを定義し、 F に付随する標準的 L -関数を具体的に決定した。更にこの池田–宮脇リフトの周期と幾つかの L -関数の臨界値の積の間に簡明な関係式が存在するとの興味深い周期予想を提出した。現在までにこの予想を肯定的あるいは否定的に支持する実例は知られていない。我々は この講演で、ある条件の下で、この予想に対する弱い意味での肯定的解決を与える。更に、この予想を肯定的に支持する2個の次数3のジーゲルモジュラー形式の実例を与える。これは伊吹山、桂田両氏との共同研究である。

3月15日(土)

3月15日, 10:00–11:00 鈴木 望夢 (東京理科大学)

題目: Calculating the index of equation orders using Newton polygons and its applications.

アブストラクト:

The gcd of the group index of equation orders in the ring of integers of a number field K is called the index of K . It is an obstruction of the integer ring to have a power basis and also is a product of primes to which Dedekind’s prime decomposition theorem cannot directly applies. Montes and Nart gave a necessary and sufficient condition that the index of the equation order coincides with the quantity derived from the Newton polygon by extending a result of Ore. However, they omitted the proof of the result, and the condition has a minor gap. In this talk, a precise form of the theorem and its application will be given.

3月15日, 11:20–12:20 臼杵 峻亮 (京都大学)

題目: On a lower bound of the number of integers in Littlewood’s conjecture.

アブストラクト:

Littlewood’s conjecture is a famous and long-standing open problem in Diophantine approximation, and is closely related to the action of diagonal matrices on $\mathrm{SL}(3, \mathbb{R})/\mathrm{SL}(3, \mathbb{Z})$ (diagonal action). From this relation and a rigidity property of the action, a breakthrough for this conjecture was made in the 2000’s. In this talk, I will explain such a relation between Littlewood’s conjecture and the diagonal action. I will also explain my result on “quantitative” Littlewood’s conjecture which is also derived from some properties of the diagonal action.

3月15日, 14:00–15:00 Joseph Muller (東京大学)

題目: Nearby cycles of the PEL $\mathrm{GU}(1, n-1)$ Shimura variety over a ramified prime.

アブストラクト:

Shimura varieties associated to groups of unitary similitudes have a classical description as moduli spaces of polarized abelian varieties with extra structures. In the case of signature $(1, n-1)$, Pappas built a flat integral model of this Shimura variety over primes which ramify in the reflex field. This model has isolated singularities located inside the special fiber. In this talk, I will explain how to compute the cohomology sheaves of the ℓ -adic nearby cycles complex of this integral model. Our computations rely on Krämer’s explicit description of the local model and of its blow-up at the singular points. Since the blow-up has semi-stable reduction, the nearby cycles can now be computed by proper base change.

3月15日, 15:20–16:20 藤井 俊 (島根大学)

題目: On families of imaginary abelian fields with pseudo-null unramified Iwasawa modules.

アブストラクト:

Let p be a prime number and k a finite extension of \mathbb{Q} . Let K/k be the maximal multiple \mathbb{Z}_p -extension and X the Galois group of the maximal unramified pro- p abelian extension L_K/K . Then the complete group ring $\Lambda = \mathbb{Z}_p[[\mathrm{Gal}(K/k)]]$ acts on X , and it is known that X is finitely generated and torsion over Λ . Further, it is conjectured by Greenberg that X is pseudo-null over Λ . In this talk, for each prime number p , we shall show that there are infinitely many abelian fields k such that $X \neq 0$ and that X is pseudo-null.

3月15日, 16:40–17:40 足立 恒雄 (早稲田大学)

題目: The fundamental Equality for Semi-discrete Valuations.

アブストラクト:

Let v be a semi-discrete valuation of a field K , which indicates that the valuation group of v is isomorphic to the finite direct sum of the integers with lexicographic order. Let L be a finite separable extension of K . We study the conditions under which the fundamental equality of the extensions of v to L holds.